Econ 802

Second Midterm

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All questions have equal weight. If anything is unclear, please ask.

1. For fixed input prices $(w_1, w_2) > 0$, assume there is a unique input bundle $(x_1, x_2) > 0$ that minimizes cost for each output level $y > 0$. The expansion path is the set of all such input bundles, corresponding to all possible outputs $y > 0$.

(a) Prove that if the production function $y = f(x)$ is homothetic, the input ratio x_1/x_2 is identical at all points on the expansion path. Explain your result using a graph.

(b) Can a homogeneous production function (of any degree) give a U-shaped short run average cost curve? Justify your answer.

(c) Can a homogeneous production function (of any degree) give a U-shaped long run average cost curve? Justify your answer.

2. George has the linear utility function $u = ax_1 + bx_2$ where $a > 0$, $b > 0$, and $x \geq 0$.

(a) Solve for the indirect utility function and the expenditure function. Hint: consider the cases $p_1/p_2 > a/b$ and $p_1/p_2 < a/b$ separately and use a graph.

(b) Assume $p_1/p_2 > a/b$. Use Roy's Identity to obtain the Marshallian demands, and use Shephard's Lemma to obtain the Hicksian demands. Then use a graph and the basic identities of consumer theory to explain the relationship between these results.

(c) Initially the price of good 1 is so high that George doesn't buy any of this good. Let the initial prices and income be (p^o, m^o) and let the resulting utility be u^o. Now the price of good 1 falls and George buys a positive amount of good 1 but none of good 2. Let the new prices be $p' = (p_1', p_2^o)$. The price of good 2 does not change. How much income m' does George need at the prices p' to be exactly as well off as he was before? Is this more or less income than m^o? Explain.

3. Jane has strictly convex indifference curves, she always spends her entire income on two goods, her consumption bundles always have positive levels of both goods, and both of the goods are normal.

(a) Initially Jane faces prices p^o, has income m^o, and chooses bundle x^o. Call this point A. Now there is a discrete price increase $\Delta p_1 > 0$ (assume p_2 and m stay constant). Let the total effect on good 1 be Δx_1 and call the new bundle point C. On a graph, divide this total effect into income and substitution effects, where the substitution effect is defined by minimizing the expenditure (m') needed to reach the old utility level (u^o) at the new prices (p'). Label this intermediate point as B on your graph.

(b) Write down an algebraic equation of the form $\Delta x_1 = \Delta x_1^s + \Delta x_1^m$ where Δx_1^s is the substitution effect and Δx_1^m is the income effect. Do this using only the Marshallian demand functions, and indicate the prices and income at which each expression is evaluated. Then rewrite the substitution effect Δx_1^s so it involves only the Hicksian demand functions, and indicate the prices and utility level at which each expression is evaluated.

(c) Divide both sides of your last equation in (b) by Δp_1, let Δp_1 approach zero, and show that you get the calculus version of the Slutsky equation. Carefully explain each step in your reasoning. If you can't do every step, do as much as you can.

4. Consider the utility function $u[w(x), z]$ where $x = (x_1, x_2) \geq 0$, z is a non-negative scalar, $w(x) = \min \{ax_1, bx_2\}$ with $a > 0$ and $b > 0$, and u is an increasing function of w and z. The budget constraint is $p_1x_1 + p_2x_2 + qz = m$.

(a) In order to maximize overall utility, it is necessary to maximize $w(x)$ subject to the constraint $p_1x_1 + p_2x_2 = m^x$ where m^x is expenditure on the x goods. Derive the indirect utility function $v(p,m^x)$ for this problem. Then define a composite good X for the problem max $u(X,z)$ subject to $rX + qz = m$, and explain how to define the price r for the composite commodity X.

(b) Assume $m = 1$. Suppose you are given an arbitrary bundle $(X^*, z^*) > 0$ where X^* is the composite commodity in (a), and you want to find a price vector (r^*, q^*) such that this bundle is an optimal choice. How would you choose the prices? Explain.

(c) Suppose there are many consumers $i = 1 \,..\, n$ who all have the same utility function $u(X,z)$ but different incomes m_i. Give one example of a functional form for $u(X, z)$ such that the market demands for X and z depend <u>only</u> on aggregate income $M = m_1 + \,..\, + m_n$ and not the distribution of this income among the individual consumers. Justify your answer mathematically and show that your choice of $u(X,z)$ leads to an indirect utility function of the Gorman form.